Math Concepts for Science

Math for Science webpages originally created by

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Math is a foundation for all other sciences: Allied Health, Astronomy, Biology, Chemistry, Geology, Nursing, and Physics. If you are going into any field requiring a science background, you need certain strong math skills to be successful. This booklet covers five common math applications in sciences:

- Metric conversions
- Percentages
- Rounding
- Significant figures
- Scientific notation

See the *Math for Allied Health* booklet for information and practice with allied health math concepts.

Contents

Metric System Basics

You need to know Metric System (System Internationale, or SI) Base Units to understand metric-to-metric or metric-to-English measurement conversions:

Measurement	Base Unit	Symbol
Length	Meter	m
Mass	Gram	g
Volume	Liter	L *

* Liter is symbolized with a capital "L" to avoid confusion with the numeral "1."

The basic metric units are not always convenient for all measurements. For example, the meter (about the length of a yardstick) is not convenient for measuring microscopic objects such as bacteria. Nor is it convenient for measuring large amounts such as the distances between planets. We have to MULTIPLY or DIVIDE the base unit by multiples of ten to express much larger or smaller measurements.

The larger and smaller measurements are named with a prefix for the multiple of ten plus the base unit. The multiples are named like this:

Prefix	Mega-	Kilo-	Hecto-	Deka-	Base Unit	Deci-	Centi-	Milli-	Micro-
Means	10^6	1000	100	10	1	1/10	1/100	1/1000	$1/10^6$
Symbol*	Mm	km	hm	dam	m	dm	cm	mm	mcg or µm**

* The symbols given are for megameters, etc. For gram- or liter-based measures, substitute g or L for m
** µ is the Greek letter mu, pronounced "myoo." It is used as a symbol for "micro."

Remember the prefixes with the following acronyms:

Mighty King Henry Died Badly; Don't Call Me, Molly
or
Miss Kathy Has Dirty Underwear; Don't Check Mine, Mister
or
Many Kangaroos Hop Down My Driveway Carrying M & Ms

Try making up your own acronym!

Metric-Metric Conversions

To convert from one metric unit to another, ask yourself:

What unit am I starting with?	Step 1: write the given value & units on the left side of the paper
What unit do we want to end with?	Step 2: write the asked for units on the right side of the paper
And how are we going to get there?	Step 3: apply a conversion factor(s) to convert the units in the given value to the units in the answer. Put this in the middle and multiply with starting unit to get answer

Problem: Convert 3.56 meters (m) to millimeters (mm)

Solution:

What unit am I starting with?	Meters (m)
What unit do I want to end with?	Millimeters (mm)
How am I going to get there?	What unit factor relationship do I know between meters and mm? 1000 mm = 1 m
How can I express this as a fraction?	1000 mm/1 m or 1 m/1000 mm
Which fraction will I pick?	I will pick the one with the m on the bottom so that it will cancel with what I was given.

$$3.56\cancel{m}\times\frac{1000mm}{1\cancel{m}}=3560mm$$

ALWAYS include the units as you work problems. The units will help you decide how to place the conversion factor. Your goal is to set up the problem so that the units you don't want are cancelled out, and the units you have been asked for are in the answer.

Let's try another one! This one is more complicated because it doesn't use a base unit. When a problem doesn't use a base unit, first convert your given quantity to a base unit, and then do a second conversion to the unit you want.

Problem: Convert 7980 micrograms (µg) to milligrams (mg)

Solution:

What unit am I starting with? Micrograms (µg)

What unit do I want to end with? Milligrams (mg)

How am I going to get there? What unit factor relationships do I know between µg, g and mg?

10^6 µg = 1 g and 1000 mg = 1 g

How can I express these as fractions? $\frac{10^6\ \mu g}{1\ g}$ *or* $\frac{1\ g}{10^6\ \mu g}$ and $\frac{1000 mg}{1g}$ or $\frac{1\ g}{1000\ mg}$

Which fractions will I pick? Look at the given units. Your **first** conversion factor must have these units on the bottom so they will cancel.

Now which unit is on the top? Your **second** conversion factor must have these units on the bottom so they will cancel.

Now which unit is on the top? **These should be the units you want in your answer!!**

$$7980\cancel{\mu g} * \frac{1\cancel{g}}{10^6\cancel{\mu g}} * \frac{1000mg}{1\cancel{g}} = 7.980mg$$

Starting Amount, with UNITS	Conversion Factor(s)	Final Amount, with UNITS
Write down the amount you're given	1. Decide which conversion factors to use. 2. Working from left to right, multiply the "tops" and divide by the "bottoms."	Your answer!

PROBLEM: Convert 425 mg to cg.

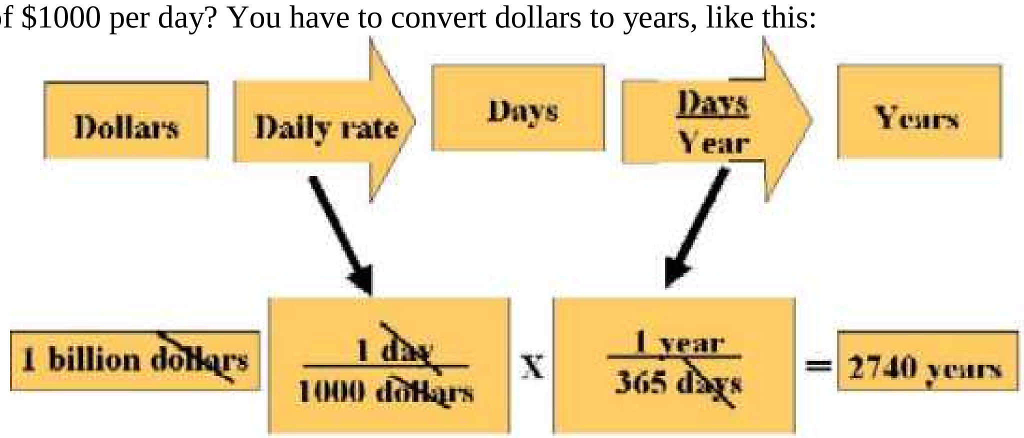
Starting Amount, with UNITS	Conversion Factor(s)	Final Amount, with UNITS
425 mg	$\frac{1\cancel{g}}{1000\cancel{mg}} * \frac{100cg}{1\cancel{g}}$	42.5 cg

PROBLEM: Convert 574 µm to mm.

Starting Amount, with UNITS	Conversion Factor(s)	Final Amount, with UNITS
574 µm	$\frac{1\cancel{m}}{10^6\cancel{\mu m}} * \frac{1000mm}{1\cancel{m}}$	0.574 mm

When a problem doesn't use a base unit, first convert your given quantity to a base unit, and then do a second conversion to the unit you want.

Suppose you had a billion dollars (watch out Bill Gates!). How many years would it take you to spend it at the rate of $1000 per day? You have to convert dollars to years, like this:



Notice that the "dollar" and "day" units cancel each other, so you are left with an answer in "years."

Metric to Metric Conversion Practice Problems

1. 360 mL to L
2. 256 mg to dg
3. 230 mm to µm
4. 14.3 L to daL
5. 37,521 m to km
6. 4532 cm to m
7. 480 kg to Mg
8. 150 daL to dL
9. 342 hm to km
10. 49,705 L to mL
11. 67 km to cm
12. 74.534 kg to g
13. 0.234 L to cL
14. 0.004732 m to cm
15. 798,567 mg to kg
16. 0.574 Mm to dam
17. 0.035 daL to µL
18. 1.865 hm to cm
19. 8650 mcg to dg
20. 0.0000678 L to mL

See answers in appendix p. 22

English-Metric Measurement Conversions

Even though science and most of the rest of the world use the metric system, here in the United States we still use the so-called "English" system of weights and measures. In some medical fields, you even work with old apothecary measures like "drams" and "grains"! How do we convert measurements from one system to another?

Equivalent Metric-English Measures

UNIT	ENGLISH to METRIC	METRIC to ENGLISH
Length	1 mile = 1.61 kilometers 1 yard = 0.914 meter 1 foot = 0.305 meter 1 inch = 2.54 centimeters	1 kilometer = 0.621 mile 1 meter = 1.09 yard 1 meter = 3.28 feet 1 centimeter = 0.394 in
Volume	1 gallon = 3.79 liters 1 quart = 0.946 liter	1 liter = 0.264 gallon 1 liter = 1.06 quarts
Mass/Weight	1 pound = 0.454 kilogram 1 ounce = 28.35 grams	1 kilogram = 2.2 pounds 1 gram = 0.0353 ounce

To convert from one unit to another, ask yourself:

what unit am I starting with? Step 1: write the given value & units on the left side of the paper

what unit do I want to end with? Step 2: write the asked for units on the right side of the paper

and how am I going to get there? Step 3: apply a conversion factor(s) to convert the units in the given value to the units in the answer. Put this in the middle and multiply with starting unit to get answer

Problem: Convert 3.56 gallons (gal) to milliliters (mL)

What unit am I starting with? Gallons (gal)

What unit do I want to end with? Milliliters (mL)

How are we going to get there? What relationships do I know between gallons and mL?
1 gal = 3.79 L, and 1L = 1000 mL

How can I express these as fractions? 1 gal/3.79 L or 3.79 L/1 gal, and 1000 mL/1 L or 1 L/1000 mL

Which fraction will I pick? I will pick the one with the gal on the bottom, and the one with L on the bottom, so that they will cancel with what I was given.

$$3.56\,\cancel{gal} * \frac{3.79\,L}{1\,\cancel{gal}} = 3.56 * 3.79\,L + 13.4924\,L \qquad 13.49\,\cancel{L} * \frac{1000\,mL}{1\,\cancel{L}} = 13.49 * 1000\,mL = 13{,}492.4\,mL$$

ALWAYS include the units as you work problems. The units will help you decide how to place the conversion factor. Your goal is to set up the problem so that the units you don't want are cancelled out, and the units you have been asked for are in the answer.

Besides metric measurements like *cc* and English measurements like *teaspoon*, nursing and other allied health sciences use some unusual units of measurement from the apothecary system.

Why these odd units? Because medicine is an ancient profession, and tradition dies hard. These apothecary units are not common anymore, but you need to know them.

UNIT	**Equivalents**	**What they Mean**
minim	1/60 dr	from the Latin *minima*, meaning smallest or least
dram (dr)	60 grains, or 3.887 g	from the Greek *drachma*, a small coin
grain (gr)	1/60 dr or 0.0648 g or 60mg 15 gr = 1 g	from the Latin *granum*, grain or seed; based on the weight of one grain of wheat
drop (gtt)	Drops come in two sizes, macro and micro, depending on the type of IV tubing. 1 mL= 10-20 macrogtts (mgtts); (15macrogtts may be standard) 1 mL = 60 microgtts (µgtts)	"gtt" is from the Latin *gutta*, meaning a drop; µ is the Greek letter *mu*, pronounced "myoo." It is used as a symbol for "micro."

English to English	**English to Metric**
4 quarts (qt) = 1 gallon (gal) 1 qt = 2 pints (pt) 1 pt = 16 ounces (oz) or 2 cups 1 glass or 1 cup = 8 oz 1 oz = 2 Tbsp = 6 tsp	1 oz = 30 mL 1 tsp = 5 mL 1 Tbsp = 15 mL

Counting calories? 1 g CHo (carbohydrate) OR 1 g protein = 4 cal; 1 g fat = 9 cal

PRACTICE PROBLEMS Convert the following English measures of volume, length, and weight to their metric equivalents. Watch units carefully!

1. 5 gallons to liters
2. 2 gallons to mL
3. 3 quarts to liters
4. 1 1/2 quarts to mL
5. 3 miles to km
6. 4.7 yards to meters
7. 2.6 feet to meters
8. 5 feet to meters
9. 118 pounds to kg
10. 24 lbs. to kg
11. 16 oz. to g
12. 5 oz. to g

See answers in appendix p. 22

Percents

"I got a 7% raise!" *George got a 67% on his calculus exam.*

Fredricka is making only 23.3% of her free throws; Sally is making 34%.

The rain forest is vanishing at a rate of 5% per year. *This spaghetti sauce is 100% organic.*

High tech stocks gained 15.2% this quarter, while utilities fell 4.25%.

Your diet should have 60% of total calories from carbohydrates,
10% from protein, and less than 30% from fats

Percent (symbol %) means "per one hundred," from the Latin *per centum*. Use 100% to mean a whole, and figures such as 25% to mean parts of a whole. Because percents are based on 100, they are easier to calculate, visualize, relate, and compare than fractions. Percents are commonly used to be a standard for comparison, to compare parts to a whole, and to compare parts of different wholes.

Percents are another way to express fractions, decimals, and ratios. Because percent is expressed as a number in relation to 100, it is easier to use it to compare what otherwise might seem very dissimilar or confusing figures.

For instance, 18 out of 82 (18/82) students passed a test in one class (it was a really, really hard test!) In another class 20 out of 75 (20/75) passed. Which class had a higher percentage of students pass?

You can solve this problem several ways. First, convert the fractions to decimals, then the decimals to percents. To convert a decimal to a percent, move the decimal point two places to the right and add the percent sign.

$18 \div 82 = .219 = 21.9\%$ $\qquad 20 \div 75 = .266 = 26.6\%$

Or, use ratio:

$$\frac{18}{82} = \frac{n}{100}, so\ n = \frac{18 * 100}{82} = 21.9\% \qquad \frac{20}{75} = \frac{n}{100}, so\ n = \frac{20 * 100}{75} = 26.6\%$$

It is much easier to see that 26.6% > 21.9% than that 20/75 > 18/82

Here’s a visual example to demonstrate comparison with percent.

Which figure has a greater portion shaded blue, A or B?

A. 

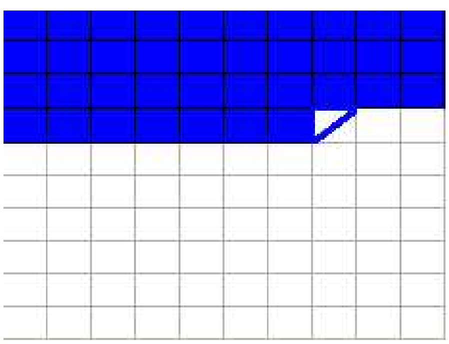
B. 

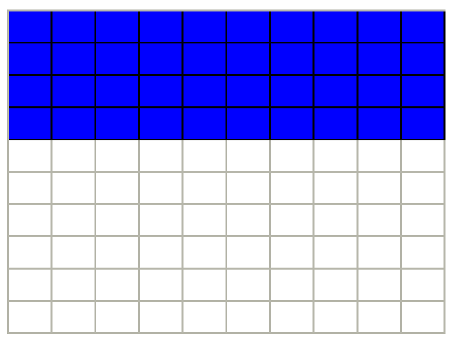
It's easy to say that 3/8 of Figure A is shaded and 4/10 of Figure B is shaded; converting the fractions to percents through ratios makes the comparison easier to visualize and quantify.

$$\frac{3}{8}=\frac{n}{100},\ so\ n=\frac{3*100}{8}=37.5\% \qquad \frac{4}{10}=\frac{n}{100},\ so\ n=\frac{4*100}{10}=40\%$$

Because 40% > 37.5%, Figure B has a 2.5% greater portion shaded than Figure A.

Here is another way to visualize the problem. Each of these boxes has 100 squares. Which one has a greater number shaded or bordered blue, A or B?

A. 

B. 

In both cases, it's easier to see that 40 > 37.5 (or 40% > 37.5%) than to see that 4/10 >3/8.
So Figure B has a 2.5% greater portion shaded than Figure A.

Percents in Chemistry Courses

Using Percent as a Unit Factor

Percent can be expressed as number of parts per hundred parts.

For example, 13% is the same as 13 parts per 100 parts, or 13/100

Notice that this is a unit (conversion) factor. Remember, unit factors have a top (numerator) and a bottom (denominator) that can be flipped as desired to work a problem.

Chemistry often uses percent in problems of mass and volume. Here are some tips for setting up and solving these problems.

See **Metric-Metric** conversions on page **3** if you need help with unit factors or help with setting up a conversion problem.

Tip 1: Determine Which Units to Use When Given a Percent

Problem: a 10.7 g sample of an unknown ore is 13.6% gold. Find the mass of gold (Au).

Solution: Read the sentence carefully. The percent (13.6% gold) can have any units you want. Since the sample is in grams, we will use grams.

The unit factor is $\frac{13.6\ g\ Au}{100\ g\ sample}$

This unit factor can be flipped in whichever direction we want (cross-canceling units) to work the problem.

$$10.7\ \cancel{g\ sample} \times \frac{13.6\ g\ Au}{100\ \cancel{g\ sample}} = 1.46\ g\ Au$$

Problem: Table salt is composed of 60.7% chlorine. Calculate the mass of chlorine in 0.462 g salt.

Solution: The unit factor is $\frac{60.7\ g\ Cl}{100\ g\ salt}$

Therefore

$$0.462\ \cancel{g\ salt} * \frac{60.7\ g\ Cl}{100\ \cancel{g\ salt}} = 0.280\ g\ Cl$$

Problem: Here's a similar problem from the other direction. Table salt is composed of 60.7% chlorine.

Calculate the mass of salt containing 1.75 g chlorine.

Solution: The unit factor is $\frac{60.7\ g\ Cl}{100\ g\ salt}$

Therefore $$1.75\ \cancel{g\ Cl} * \frac{100\ g\ salt}{60.7\ \cancel{g\ Cl}} = 2.88\ g\ salt$$

The conversion factor is flipped to cross cancel the grams Cl. We want the answer in grams salt.

Problem: A gas contains 11.6% nitrogen (symbol: N). What volume of the gas contains 36.7 L N?

Solution: The unit factor is $\frac{11.6\ L\ N}{100\ L\ gas}$

Therefore $$36.7\ \cancel{L\ N} * \frac{100\ L\ gas}{11.6\ \cancel{L\ N}} = 316\ L\ gas$$

Remember

- Your percent unit factor can be expressed in any units per 100 units.
- When you start a problem, DO NOT start with the percent!
- Always start with the other piece of data given in the problem

Tip #2 Calculate a Mass from a Given Percent

Problem: A 8.620 g sample of bauxite was found by analysis to contain 4.10% aluminum (symbol: Al). Find the mass of Al in the sample.

Solution: $$8.620\ g\ bauxite * \frac{4.10\ g\ Al}{100\ g\ bauxite} - 0.353\ g\ Al$$

Problem: A moon rock is analyzed and found to contain 19.0% iron. What is the total mass of the sample rock if 0.254 kg of iron is found?

Solution: $$0.254\ \cancel{kg\ Fe} * \frac{100\ kg\ sample}{19.0\ \cancel{kg\ Fe}} = 1.34\ kg\ sample$$

Tip #3 Given the Percent, Calculate the Total Mass from the Mass of a Constituent.

Problem: Air is 20.9% oxygen by volume. Find the volume of air that contains 631 mL of oxygen.

Solution: $$631\ mL\ O * \frac{100\ mL\ air}{20.9\ mL\ O} = 3019\ mL\ air,\ or\ 3.019\ L\ air$$

ROUNDING OFF NUMBERS

In scientific measuring or calculating, your answers will be expressed in **significant figures** (see page 15.) Measurements and calculations are either demonstrating a physical reality or are meaningful to you as a scientist. You might be given specific directions about how many significant figures you should use. You need to know the rules of rounding off numbers.

For each example, you are directed to round off to 3 significant figures.

1. If the first extra digit is LESS than 5—drop it. Now the last digit of the number remains the same.

   Ex. 4.321 becomes 4.32

2. If the first extra digit is 5 or MORE than 5, drop it and increase the last significant digit by 1.

   Ex. 4.336 becomes 4.34

SPECIAL NOTE FOR COURSES IN CHM 111 AND HIGHER: If the first "extra" digit is 5 followed by zeros, drop the 5 and EITHER increase the last *odd* significant digit by 1—e.g. 4.3350 becomes 4.34 OR leave the last *even* significant digit the same—e.g. 4.3250 becomes 4.32

3. Non-significant digits to the left of the decimal point are NOT discarded but are replaced by ZEROS

   Ex. 4321.2 becomes 4320

In the examples below, we are rounding to THREE (3) significant figures.

If the first "extra' digit is:	Then do this:
Less than 5 For example: 4.321	Drop it. Now the last digit of the number remains the same. For example: 4.32
5 or more For example: 4.336	Drop it and INCREASE the last significant digit by 1. For example: 4.34
A placeholder and NOT a significant figure For example: 4321.2	Do NOT discard it; replace it with a zero (0). For example: 4320
Special Note for Chemistry classes from CHM111 on up: When the first extra digit is 5 followed by zeros, drop the **5 and** the zeros. For example: 4.3***350*** to 4.3***3*** 4.3***250*** to 4.3***2***	THEN, if the last significant digit is odd, increase it by 1. For example: 4.3***4*** If the last significant digit is even, leave it the same. For example: 4.3***2***
Special Note for **Nursing Math**: follow these tips for rounding in flow rate problems.	

Practice Problems in Rounding

1. Round 56.84251 to the nearest: a) tenth b) hundredth c) thousandth

2. Round 97.26 to the nearest tenth

3. Round 0.11826 to the nearest a) thousandth b) tenth

4. Round 0.8834 to the nearest hundredth

5. Round 11344 to the nearest a) thousand b) hundred

6. Round 345 to nearest ten

7. Round to the nearest thousand a) 99371 b) 85643 c) 1278

8. Round to the nearest hundredth using the special CHM 111 rule regarding 5 or more.

   A) 126.23500
   B) 1.0450

See answers in appendix on page 22.

SIGNIFICANT FIGURES

You need precision and accuracy in measurement and calculation for science. When measurement numbers are gathered or calculated, to **be** precise and accurate you must determine which digits in the numbers are necessary—-that is, which are the **significant figures**.

When reading a measurement someone else has made, it is important that you recognize how many significant figures are present. The number of significant figures indicates the accuracy of the measurement.

The rules for determining these ***significant figures*** in measurement are:

- ***Non-zero digits*** are ALWAYS significant: for 487, all three digits are significant.
- Any ***zeroes between 2 significant figures*** are significant: for 304, all digits are significant.
- After a decimal point, ***any zeroes to the right of significant figures*** (even added at the end) are significant: for 34.57000, all seven digits are significant
- ***All*** digits in scientific notation are significant. See Scientific Notation on page 18.

To determine the number of significant figures in a measurement:

- Start with the ***first non-zero digit on the left***
- Count ***all*** the digits from the first non-zero digit to the end of the number
- ***Non-zero digits*** are ALWAYS significant
- Any ***zeroes between two (2) non-zero digits*** are significant
- Any ***zeroes added at the end after a decimal point*** are significant
- ***All*** digits in scientific notation are significant. See Scientific Notation on page 18.

When you apply the rules, you find that

- 1.498 g contains 4 significant figures-- 1.498 g
- 209 mm contains 3 significant figures-- 209 mm
- 0.00097 mg contains 2 significant figures-- 0.00097 mg
- 0.18700 mL contains 5 significant figures-- 0.18700 mL
- 2.1860 cc contains 5 significant figures-- 2.1860 cc
- 498.00 m contains 5 significant figures-- 498.00 m
- 3.68×10^6 contains 3 significant figures-- 3.68×10^6

How to Determine Which Figures Are Significant

The Digits	Digits That Count	Example	Number of Significant Figures
Non-zero digits	ALL	4.337	4
Leading zeros (zeros at the BEGINNING)	NONE	0.00065	2
Captive zeros (zeros BETWEEN non-zero digits)	ALL	1.000023	7
Trailing zeros (zeros at the END)	ONLY IF they follow a significant figure AND there is a decimal point in the number	89.00 but 8900	4 2
Leading, Captive AND Trailing Zeros	Combine the rules above	0.003020 but 3020	4 3
Scientific Notation	**ALL**	7.78 x 10^3	3

Significant Figures in Calculations

Calculations use two or more numbers which may contain different amounts of significant figures.

When **multiplying** or **dividing**, this rule applies: Your answer may contain ***no more*** significant figures than the number used in the calculation that has the ***fewest*** sig figs.

FOR EXAMPLE: .45cm x 1.336 = 0.6012
.45 has only **2** significant figures, so the answer should be 0.60 cm

When **adding** or **subtracting**, find the number that has the ***fewest*** digits to the ***right*** of the decimal point. The answer must contain no more digits to the RIGHT of the decimal point than that number in the problem. FOR EXAMPLE:

5.52 g 4.1 g +3.238 g 12.858 g rounded to 12.9 g	The sum is 12.858 g, with three digits to the right of the decimal point. Since the measurement 4.1 g in the problem has the ***fewest*** digits to the ***right*** of the decimal point, your answer should have only 1 digit after the decimal point. **So round the answer to *12.9 g***

Examine the Problem	Example
When you **multiply** or **divide** two or more numbers, find the one that has the ***fewest*** significant figures. That's how many significant figures should be in your answer	**3.35 x 4.669 mL = 15.571115 mL** **rounded to 15.6 mL** 3.35 has only **3** significant figures, so that's how many should be in the answer. Round it off to **15.6mL**
When you **add** or **subtract** two or more numbers, find the one that has the ***fewest*** digits to the ***right*** of the decimal point. The answer must contain **no more digits** to the **RIGHT** of the decimal point than the number in the problem.	**64.25 cm + 5.333 cm = 69.583 cm** **rounded to 69.58 cm** 64.25 has only **two** digits to the right of the decimal, so that's how many should be to the right of the decimal in the answer. Drop the last digit so the answer is **69.58 cm.**

PRACTICE PROBLEMS WITH SIGNIFICANT FIGURES

1. How many significant figures are in these measurements?

   a. 32 cm.
   b. 34.6 g
   c. 2.489 g
   d. 53.670 mL
   e. 444.3060 km

2. Round off the following to 2 significant figures.

   A. a. 90.356 kg
   B. 0.00775 mm
   C. 12.34665 g
   D. 1.023 m

3. Express the answer to each of the following problems with the correct number of significant figures.

   a. 106.23 g + 70.0 g =
   b. 6612.335 in^2 ÷ 5.24 in^2
   c. 0.09121 mm x 11.3 mm =
   d. 33.34 mL - 1.6 mL =
   e. 0.225 mg - 0.0667 mg =
   f. 55.6 kg - 4.996 kg =
   g. 6.335 g + 0.003 g =
   h. 123.7 mm x 0.99 mm =
   i. 707.62 m ÷ 75 m =

See answers in appendix on p. 22.

SCIENTIFIC NOTATION

Many areas of science rely on expressing numbers in **scientific** or **exponential notation** when those numbers are astronomically large (such as the distances between planets) or infinitesimally small (such as the mass of an electron).

Scientific notation gives greater accuracy, concision, and ease of calculation. In **scientific notation**, a number is given as a product of a number between 1 and 10 and a power of ten. For example, 56,300 is written in scientific notation as

- 5.63×10^{4}, meaning
- 5.63 x 10 x 10 x 10 x 10
- or 5.63 x 10,000

Scientific notation ***always has only ONE nonzero digit to the left of the decimal, like this:***
5.63 (not 56.3 or 563) x 10^{4}

When to Use Scientific Notation

Express numbers in scientific notation under the following conditions.

- **A number with ambiguous zeros:** these are trailing zeros to the right in a number without a decimal. To remove the ambiguity, it MUST be expressed in scientific notation.
  - It is not clear whether 35000 has 2, 3, 4 or 5 significant figures (popularly known as sig figs; see page 15). It is not clear whether the three trailing zeros are significant or not.
  - If you mean 35000 to have 3 sig fig, then it MUST be expressed as 3.50×10^{4}.
- **A very small number** (in general, less than 0.01): It is tedious and risky to copy numbers with a string of avoidable zeros. For instance, 0.00000083 SHOULD be expressed as 8.3×10^{-7}.
- **A very large number:** Again, for greater accuracy, concision and ease of use, 1,563,000,000,000 is probably better as 1.563×10^{12}.
- **A number that is in exponential form** for any reason. For instance, 324.3×10^{-8} SHOULD be expressed as 3.243×10^{-6}.

Express numbers in scientific notation when necessary for **accuracy, concision**, and **ease of use**. Although it is not "wrong" to use scientific notation, you should learn when it is appropriate. For example, it would not be appropriate to tell someone to weigh out "2.5×10^{1} grams of salt" when "25 grams of salt" would do. But it *is* appropriate to say that a light-year is 9.5×10^{12} km rather than 9,500,000,000,000 km

Follow these steps to convert very large or small numbers to scientific notation:

1. Separate the very large or very small number into two factors:

- The first factor is the first non-zero integer of the number followed by a decimal point and the remaining integers of the number after the decimal.
- The second factor is a power of ten based on the number of integers in the original number

For example, the number 54,000 would be separated into two factors: 5.4 and 10,000

The number 54,280 would be separated into these factors: 5.428 and 10,000.

The number 0.0005428 would be separated into these factors: 5.428 and 0.0001

Move the decimal point so that it is between the first non-zero integer at the **left** end of the number and the one next to it on its right. There is always **only one integer to the left of the decimal point** in the first factor of scientific notation.

2. The two factors are then written as a product, with the second factor written exponentially as a power of ten:

$$5.4 \times 10^{4}$$
$$5.428 \times 10^{4}$$
$$5.428 \times 10^{-4}$$

Figure out the power of ten to use by counting the number of digits you moved the decimal point. In all three of these examples, it moved **four** places.

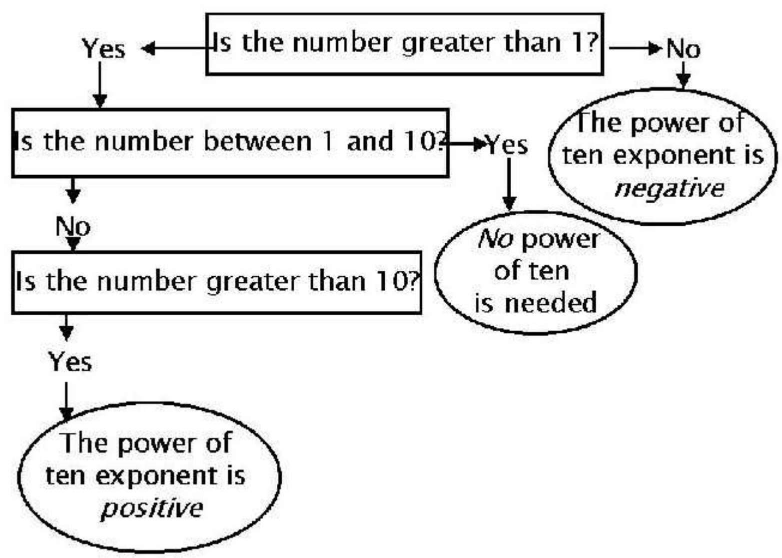
- If the original number is **greater than 1** the exponent is **positive.**
- If the original number is **between 0 and 1**, the exponent is **negative.**

$10^{4} = 10 \times 10 \times 10 \times 10 = 10000$
$10^{3} = 10 \times 10 \times 10 = 1000$
$10^{2} = 10 \times 10 = 100$
$10^{1} = 10$
$10^{0} = 1$
$10^{-1} = 1/10 = 0.1$
$10^{-2} = 1/100 = 0.01$
$10^{-3} = 1/1000 = 0.001$
$10^{-4} = 1/10000 = 0.0001$

In the examples given above,

54,000 would be expressed as	5.4 x 10,000	or as	5.4 x 10^4
54,280 would be expressed as	5.428 x 10,000	or as	5.428 x 10^4
0.0005428 would be expressed as	5.428 x 0.0001	or as	5.428 x 10^{-4}

Scientific Notation Flow Chart



- Create the first factor by moving the decimal point of the number to the right or left to create a number with only one digit to the LEFT of the decimal.
- Count the number of digits you have moved the decimal place to the left or right to determine which power of ten to use.
- If the original number is *greater than 1* (X > 1), the exponent is *positive*.
- If the original number is *between 0 and 1* (0 < X < 1), the exponent is *negative*.
- By the way-- $10^{\mathbf{0}}$ = 1. *Any* number to the zero power always equals one

Number	**Multiple of Ten**	**Written as Exponent**
10	10	10^1
100	10 x 10	10^2
1000	10 x 10 x 10	10^3
10000	10 x 10 x 10 x 10	10^4
1/10 or 0.1	1/10	10^{-1}
1/100 or 0.01	1/10 x 1/10	10^{-2}

Examples

Number	Number written in expanded form--number times a multiple of ten	Number written in exponent form
20	2.0 x 10	2.0 x 10^{1}
356	3.56 x 10 x 10	3.56 x 10^{2}
4067	4.067 x 10 x 10 x 10	4.067 x 10^{3}
28766	2.8766 x 10 x 10 x 10 x 10	2.8766 x 10^{4}
0.571	5.71 x 1/10 **OR** 5.71 x 0.1	5.71 x 10^{-1}
0.093	9.3 x 1/10 x 1/10 **OR** 9.3 x 0.01	9.3 x 10^{-2}

Scientific Notation Practice

Express the following in scientific notation:

1. 6,000,000,000,000,000,000,000
2. 547,000
3. 3,507,000,000
4. 0.00000000000463
5. 0.00090674
6. 790,640,000
7. 0.00000047906
8. 98,000,000,000,000,000,000,000,000,000,000
9. 0.000000049704
10. 0.0004650

Express the following in decimal form:

1. 3.4067 x 10^{7}
2. 5.0400 x 10^{-4}
3. 4.0689 x 10^{19}
4. 1.0015 x 10^{9}
5. 8.0000 x 10^{-6}
6. 6.4523 x 10^{-5}
7. 8.213 x 10^{2}

See answers in appendix on page 23.

ANSWERS

Metric to Metric Conversions p. 6

1. 0.36 L
2. 2.56 dg
3. 230,000 µm
4. 1.43 daL
5. 37.521 km
6. 45.32 m
7. 0.480 mg
8. 15,000 dL
9. 34.2 km
10. 49,705,000 mL
11. 6,700,000 cm
12. 74,534 g
13. 23.4 cL
14. 47.32 m
15. 0.798567 kg
16. 57,400 dam
17. 350,000 µL
18. 18,650 cm
19. 0.0865 dg
20. 0.0678 mL

English to Metric Conversions p. 8

1. 18.95 L
2. 7580 Ml
3. 2.84 L
4. 1419 mL
5. 4.83 km
6. 4.3 m
7. 0.793 m
8. 1.5 m
9. 53.6 kg
10. 10.9 kg
11. 453.59 g
12. 141.75 g

Rounding Numbers p. 14

1. a. 56.8 b. 56.84 c. 56.843

2. 97.3

3. a. 0.118 b. 0.1

4. 0.88

5. a. 11000 b. 11300

6. 350

7. a. 99000 b. 86000 c. 1000

8. a. 126.24 b. 1.04

Significant Figures p. 17

1. a. 2 b. 3 c. 4 d. 5 e. 7

2. a. 90. kg b. 0.0078 mm c. 12 g d. 1.0m

3. a. 176.2 (4 sigfigs)
   b. 1260 in. (3 sigfigs)
   c. 1.03 mm^2 (3 sigfigs)
   d. 31.7 mL (3 sigfigs)
   e. 0.158 mg (3 sigfigs)
   f. 50.6 kg (3 sigfigs)
   g. 6.338 g (4 sigfigs)
   h. 120 mm^2 (2 sigfigs)
   i. 9.4 m (2 sigfigs)

Scientific Notation p. 21

Answers expressed in scientific notation:

1. 6.0×10^{21}
2. 5.47×10^{5}
3. 3.507×10^{9}
4. 4.63×10^{-12}
5. 9.0674×10^{-4}
6. 7.9064×10^{8}
7. 4.7906×10^{-7}
8. 9.8×10^{31}
9. 4.9704×10^{-8}
10. 4.650×10^{-4}

Answers expressed in decimal form:

1. 34,067,000
2. 0.00050400
3. 40,689,000,000,000,000,000
4. 1,001,500,000
5. 0.0000080000
6. 0.000064523
7. 821.3